Noise Analysis and Modeling

Noise Analysis and Modeling

- Circuit noise
  - Interference noise
  - Inherent noise

- Interference noise
  - Result from interaction between circuit and outside world or between different parts of circuit itself.
  - Examples :
    - Power supply noise on ground wires
    - Electromagnetic interference between wires
  - Can be reduced by careful circuit wiring or layout

Noise Analysis and Modeling (Cont.)

- Inherent noise
  - Refers to random noise signals that can be reduced but never eliminated since this noise is due to fundamental properties of circuits.
  - Examples :
    - Thermal noise and flicker noise
    - Only moderately affected by circuit wiring or layout, such as using multiple contact to change resistance value of a transistor. However, inherent noise can be significantly reduced through proper circuit design, such as changing the circuit structure or increasing bias current.

- Only inherent noise will be discussed in the following

Time-Domain Analysis

- Assumption : All noise signals have a mean value of zero. This assumption is valid in most physical systems.
- Root mean square(rms) voltage value is defined as

$$V_{n(rms)} = [\frac{1}{T}\int_0^T V_n^2(t)dt]^{1/2}$$

Root mean square(rms) current is defined as

$$I_{n(rms)} = [\frac{1}{T}\int_0^T I_n^2(t)dt]^{1/2}$$

Typically, a longer T gives a more accurate rms measurement

- Normalized noise power, p_{diss}

$V_n(t)$ is applied to a 1 Ω resistor

$$P_{diss} = \frac{V_{n(rms)}^2}{1\Omega} = V_{n(rms)}^2$$

or $P_{diss} = I_{n(rms)}^2 \times 1\Omega = I_{n(rms)}^2$

Time-Domain Analysis (Cont.)

- Signal-to noise ratio(SNR), dB

$$SNR = 10\log[\frac{\text{signal power}}{\text{noise power}}]$$

  - Example : normalized signal power = $V_{s(rms)}^2$

    normalized noise power = $V_{n(rms)}^2$

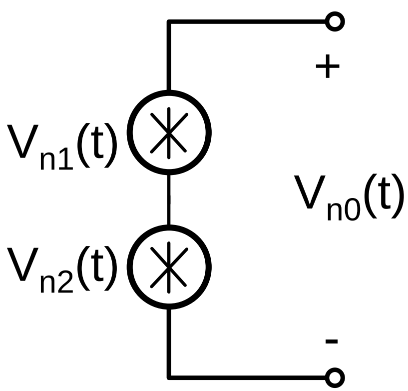
$$SNR = 10\log[\frac{V_{s(rms)}^2}{V_{n(rms)}^2}] = 20\log[\frac{V_{s(rms)}}{V_{n(rms)}}]$$

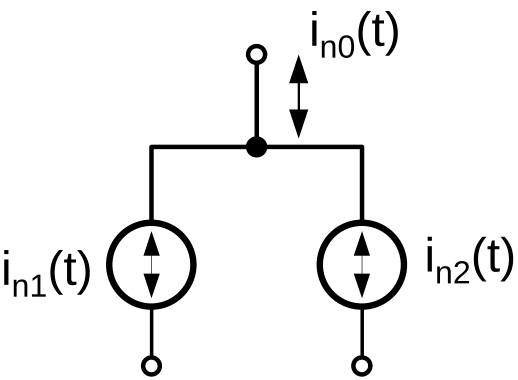
    when $V_{s(rms)}^2 = V_{n(rms)}^2$, SNR=0dB

- dBm
  - dB units relate the relative ratio of two power levels
  - For dBm units, all power levels are referenced by 1mW
    - Examples : 1mW = 0dBm and 1μW = -30dBm
  - It is common to reference the voltage level to either a 50Ω or 75Ω resistor
    - Example: $10\log\frac{V_{n(rms)}^2/50\Omega}{1mW}$ dBm or $10\log\frac{I_{n(rms)}^2\times 50\Omega}{1mW}$ dBm (50Ω → or 75Ω)

Noise Summation

- Noise sources $V_{n1}(t), V_{n2}(t), V_{n3}(t), \ldots\ldots$
  Total noise $V_{n0}(t) = V_{n1}(t) + V_{n2}(t) + V_{n3}(t) + \ldots$
- Example : summation of 2 noise sources
  - ◆ Voltage noises
  - ◆ Current noises





- $V_{no}(t) = V_{n1}(t) + V_{n2}(t)$

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T \left[V_{n1}(t) + V_{n2}(t)\right]^2 dt = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T V_{n1}(t)V_{n2}(t)dt$$

$$= V^2_{n1(rms)} + V^2_{n2(rms)} + 2CV_{n1(rms)}V_{n2(rms)}$$

Noise Summation (Cont.)

- Correlation coefficient

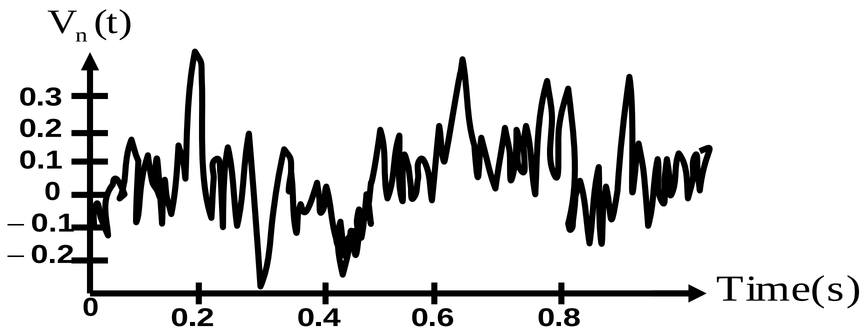
$$C = \frac{\frac{1}{T}\int_0^T V_{n1}(t)V_{n2}(t)dt}{V_{n1(rms)}V_{n2(rms)}}$$, where -1≤C≤1

  - C = ±1; the two noise signals are fully correlated
  - C = 0 ; the two noise signals are fully uncorrelated

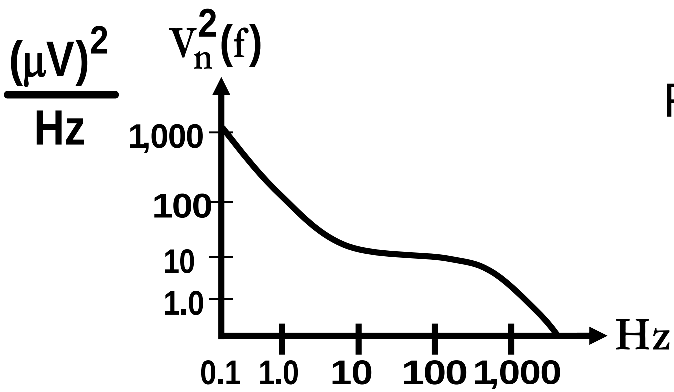
- Typically, different inherent noise sources are uncorrelated
- For two uncorrelated noise signals
  - $V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)}$
- For two fully correlated noise signals
  - $V^2_{no(rms)} = [V_{n1(rms)} \pm V_{ns(rms)}]^2$
- To reduce overall noise, concentrate on the reduction of large noise signals.

Frequency-Domain Analysis

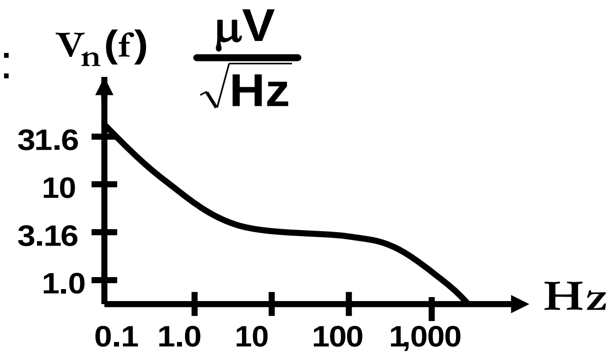
- Units of Hz (rather than radians/sec) are commonly used
- Noise spectral density
  - ◆ Periodic signals (e.g. sinusoid) have power at distinct frequency
  - ◆ Random signals have their power spread out over the frequency spectrum
- Example
  - ◆ Time-domain signal



- Spectral density



Root spectral density:



- Vertical axis is a measure of the normalized noise power over 1 Hz bandwidth at each frequency point

Frequency-Domain Analysis (Cont.)

- Resolution bandwidth (RBW)
  - V^2/Hz use 1Hz bandwidth → Normalize
  - Mean-squared value of a random signal at a precise frequency is zero.
  - Random-noise power must be measured over a specific bandwidth.

    Example1: Normalized power between 99.5Hz and 100.5Hz is $10(\mu v)^2$---shown in previous page

    Example2: Mean-squared value of noise power at 100Hz is $1(\mu v)^2$ when 0.1Hz is used

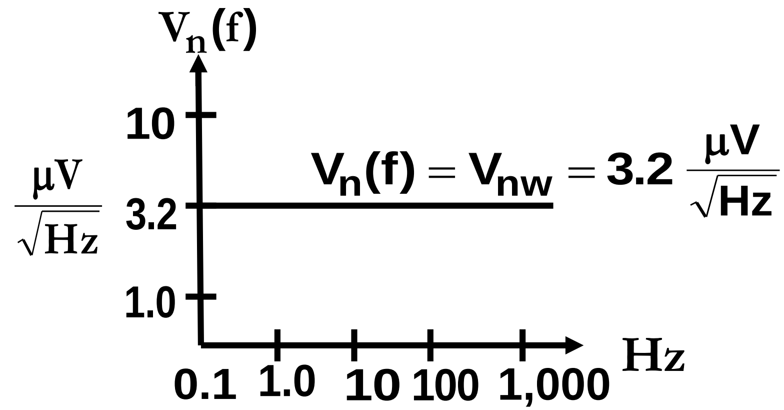
    - Mean-squared value measured at 100Hz is directly proportional to the bandwidth of the bandpass filter used for measurement

- Total mean-squared power

$$V_{n(rms)}^2 = \int_0^\infty V_n^2(f)df \qquad I_{n(rms)}^2 = \int_0^\infty I_n^2(f)df$$

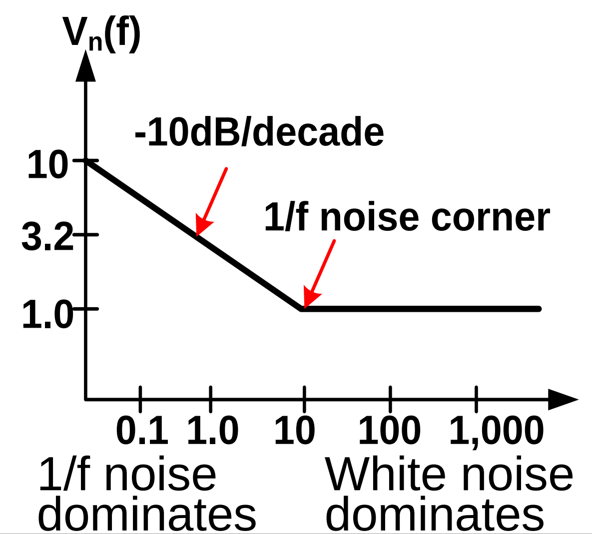
Noise Types in CMOS Transistor

- Two major sources
  - Thermal, or white noise
    flat spectrum density
    $V_n(f)=V_{nw}$ is a constant



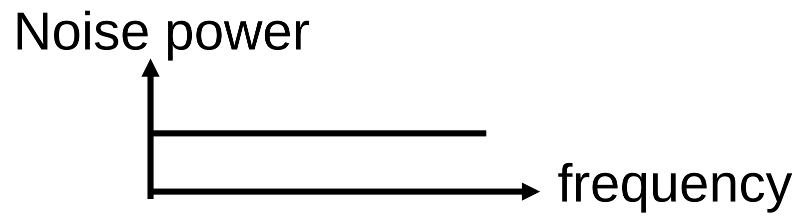
  - Flicker, or 1/f noise
    - Spectrum density is inversely proportional to frequency
      $V_n^2(f)= K_V^2/ f$ where K_v is a constant.
    - The intersection of flicker and white noise curves is called 1/f noise corner
    - Spectral density

$$V_n^2(f) \approx \frac{(3.2 \times 10^{-6})^2}{f} + (1 \times 10^{-6})^2 \quad \frac{\mu V}{\sqrt{Hz}}$$



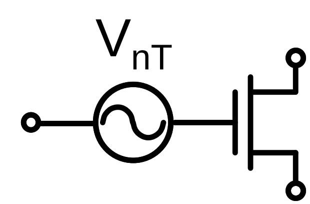
Noise in MOSFET

- Thermal noise (white noise caused by random thermal motion of electron)

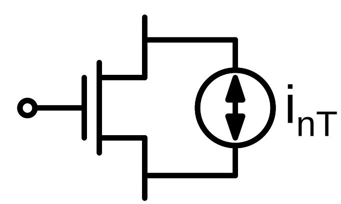


  - Real Resistor R
    mean square V_{nT}: $\overline{V_{nT}^2} = 4kTR\Delta f$
    Δf: Bandwidth in which the noise is measured, in Hz
    4kT, at room temperature, is equal to 1.66×10^{-20}VC

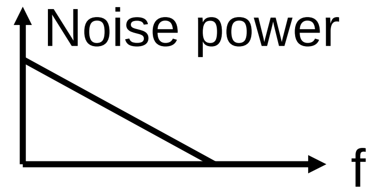
- MOSFET
  - If the device is in saturation, then $R \cong \frac{2}{3g_m}$
    - $\overline{V_{nT}^2} = \overline{\left(\frac{i_{nT}}{g_m}\right)^2} = \frac{8kT}{3g_m}\Delta f$



    Δf can't be infinite, could be assumed up to several hundred GHz for MOSFET. Since, for very high frequency ($\approx 10^{12}$Hz), other physical phenomena enter which cause $\overline{V_n^2}$ to decrease with increasing frequency



Noise in MOSFET (Cont.)

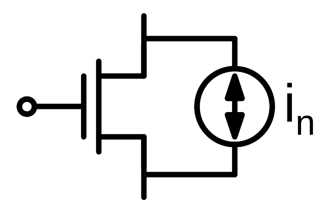


- Flicker Noise (1/f)
  In an MOS transistor, extra electron energy states exist at boundary between the Si and SiO_2. These can trap and release electrons from the channel, and hence introduce noise. Since the process is relatively slow, most of the noise energy will be at low frequency.

  $\overline{V_{nf}^2} = \frac{k}{C_{OX}WLf}\Delta f$, where k is process dependent

- Thermal noise + Flicker noise

  $$i_n = \sqrt{\overline{i_{nT}^2} + \overline{i_{nf}^2}} = \sqrt{gm^2(V_{nT}^2 + V_{nf}^2)\Delta f} = \sqrt{gm^2[\frac{8KT}{3g_m} + (\frac{K}{C_{ox}WLf})]\Delta f}$$



  where g_m is the noise conductance

- The mean squares of the noise currents are added, since the different noise mechanisms are statistically independent.

Filtered Noise

- Noise amplification and filtering
  - Spectral density

$$V_{no}^2(f) = \left|A(j2\pi f)\right|^2 V_{ni}^2(f)$$

$V_{ni}(f)$: Input noise root spectral density

$V_{no}(f)$: Output noise root spectral density

$$V_{ni}(f) \longrightarrow \boxed{A(s)} \longrightarrow V_{no}^2(f) = \left|A(j2\pi f)\right|^2 V_{ni}^2(f)$$

  - Root spectral density

$$V_{no}(f) = \left|A(j2\pi f)\right| V_{ni}(f)$$

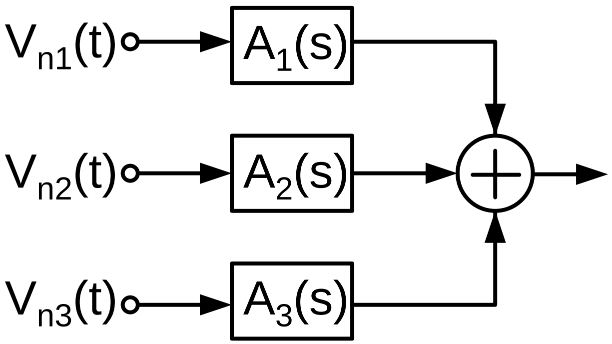
Filtered Noise (Cont.)

- The major reasons why filters are used
  - Attenuate out-of-band power
    - Avoid interference
    - Reduce signal swing and slew rate
  - Adjust in-band gain-phase relationship
- Total output mean-squared value: $V_{no(rms)}^2 = \int_0^{\infty} \left|A(j2\pi f)\right|^2 V_{ni}^2(f)df$
- Summation of multiple filtered uncorrelated noise sources

$$V_{no}^2(f) = \sum_{i=1}^{n} \left|A_i(j2\pi f)\right|^2 V_{ni}^2(f)$$

  - Example : 3 sources

Uncorrelated noise sources



$$V_{no}(f) = [\sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f)]^{1/2}$$

Noise Bandwidth

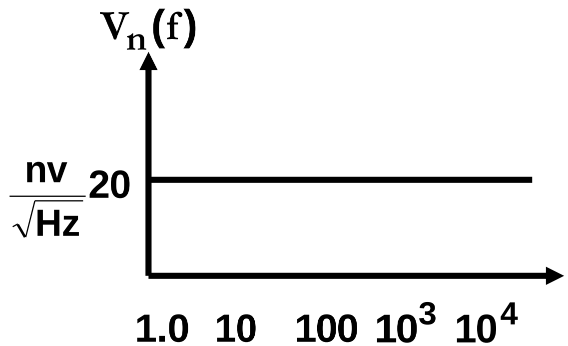
- The noise bandwidth of a given filter is equal to the frequency span of a brick wall filter that has the same output noise rms value that the given filter has when white noise is applied to both filters. (Peak gains are the same for the given and brick-wall filters.)
- Example :
  - A 1st-order lowpass response with a -3 dB bandwidth of f_o(Such a response would occur from a RC filter with $f_o = \frac{1}{2\pi RC}$)
  - Input signal $V_{ni}(f)=V_{nw}$ (White noise)
    - For the response $A(s) = \frac{1}{1+\frac{s}{2\pi f_o}}$ $\quad V^2_{no(rms)} = \int_0^{\infty} \frac{V^2_{nw}}{1+(\frac{f}{f_o})^2} df = \frac{V^2_{nw}\pi f_o}{2}$
    - For brick-wall filter with f_x bandwidth,

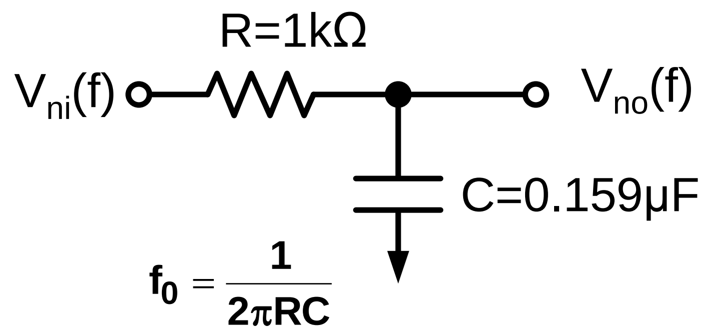
$$V^2_{no(rms)} = \int_0^{f_x} V^2_{nw} df = V^2_{nw} f_x$$

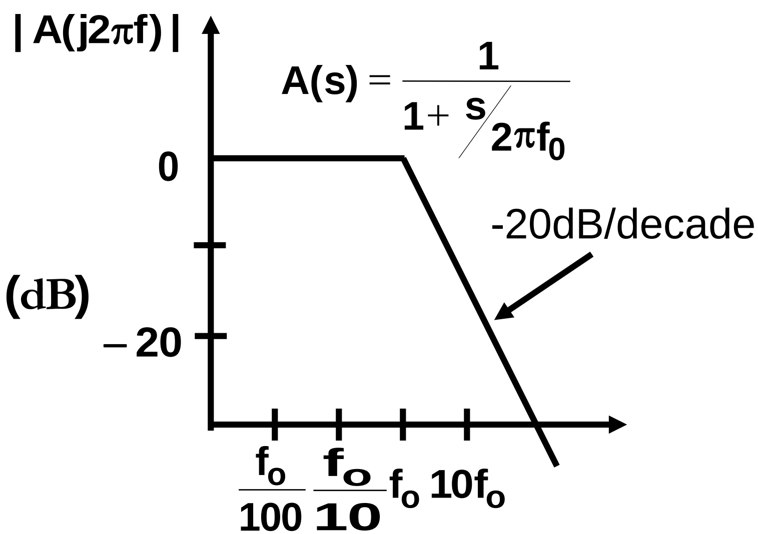
    - Therefore, noise bandwidth $f_x = \frac{\pi f_o}{2}$

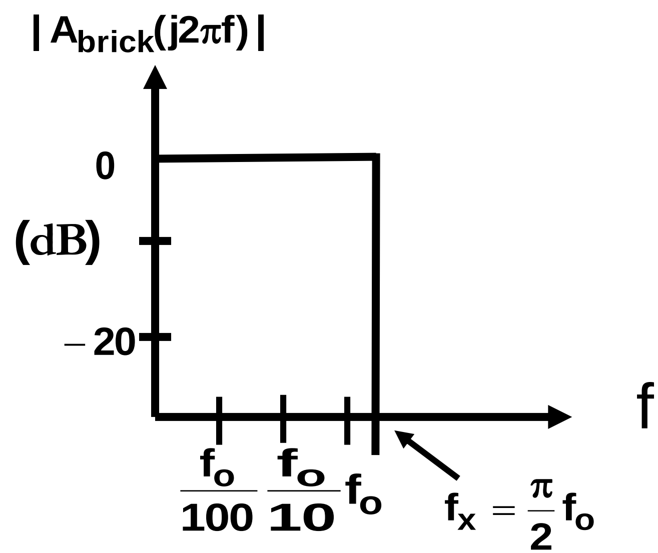
Noise Bandwidth (Cont.)

- For a RC filter $f_o = \dfrac{1}{2\pi RC}$ and $f_x = \dfrac{1}{4RC}$



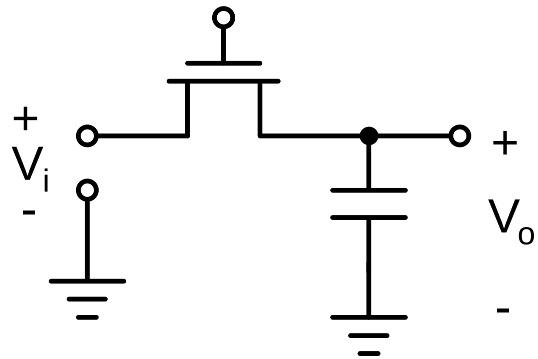




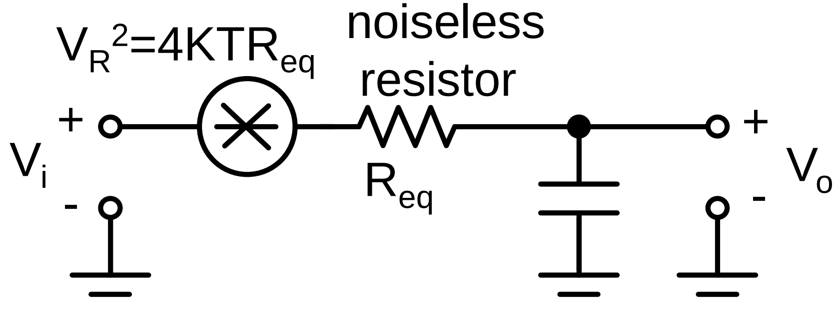


KT/C Noise

- SC sampling circuit



- Circuit noise model



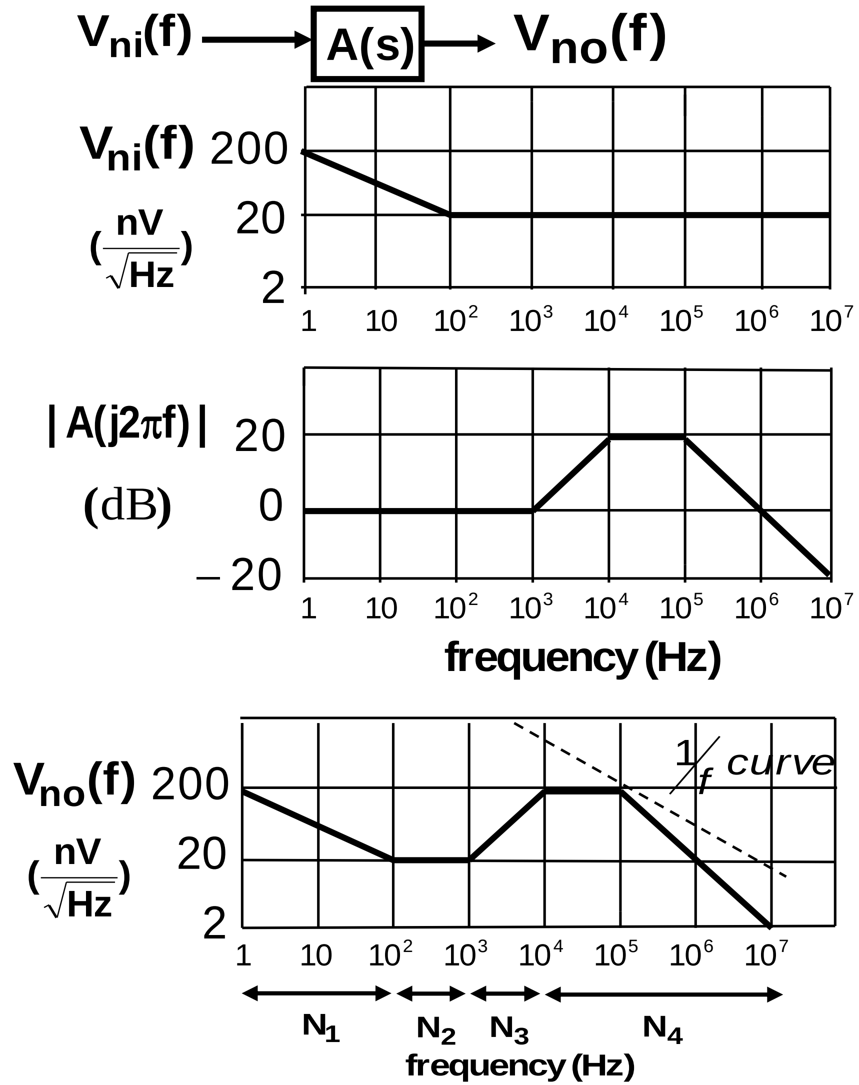
V_i=0 is assumed

$$V_{o(rms)}^2 = \frac{4KTR_{eq}}{4R_{eq}C} = \frac{KT}{C}$$

Approximate Noise Calculation

- Piecewise integration of noise
  - Simplify integration formulas
  - Integrate noise power in different frequency regions and then add together

- Example



Approximate Noise Calculation (Cont.)

$$V_{o(rms)} = \left[\int_{1}^{100} V_{no}^2(f)df + \int_{100}^{10^3} V_{no}^2(f)df + \int_{10^3}^{10^4} V_{no}^2(f)df + \int_{10^4}^{\infty} V_{no}^2(f)df \right]^{1/2}$$

$$\approx \left[1.84\times10^5 + 3.6\times10^5 + 1.33\times10^8 + 5.88\times10^9\right]^{1/2} \quad \text{(nv)}$$

- The noise power in the N_4 region is quite close to the total noise power. Thus, there is little need to find the noise contributions in N_1~N_3 regions. Such an observation leads us to the 1/f noise tangent principle.

- 1/f noise tangent principle
  - To determine the frequency region or regions that contribute to dominant noise, lower a 1/f noise line until it touches the spectral density curve --- The total noise can be approximated by the noise in the vicinity of the 1/f line.
  - The reason this simple rule works is that a curve proportional to 1/x results in equal power over each decade of frequency. Therefore, by lowering this constant power/frequency curve, the largest power contribution will touch it first.

Noise Models for Circuit Elements

- Three main noise mechanisms in transistors (BJT & MOSFET)
  - Thermal noise
    - White noise
  - Shot noise
    - Occurs in pn junctions
    - White noise
  - Flicker noise
    - 1/f noise

- Resistor noise
  - Thermal noise is the major noise source
  - Spectral density $V_R^2(f)$ or $I_R^2(f)$

Noise Models for Circuit Elements (Cont.)

- Series voltage noise source

  $V_R^2(f)=4KTR$

  where K is Boltzmamn's constant ($1.38x10^{-23}JK^{-1}$)

  T is temperature in Kelvin's

  R is the resistance value

- Parallel current noise source

$$I_R^2(f) = \frac{V_R^2(f)}{R^2} = \frac{4KT}{R}$$

- Diode noise
  - Shot noise

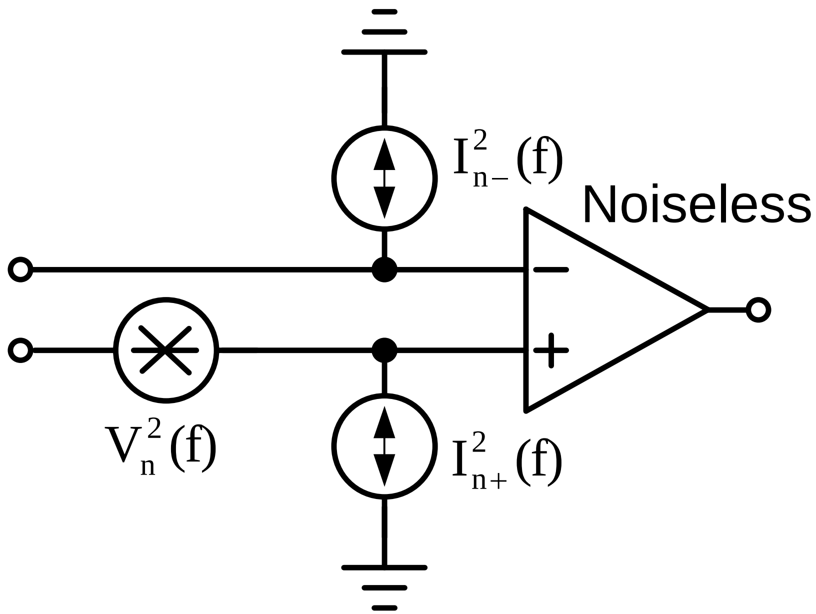
    $V_d^2(f)=2KTr_d$ where $r_d = \frac{\partial V_D}{\partial I_D} = \frac{\partial}{\partial I_D}(V_T \ln\frac{I_D}{I_S}) = V_T\frac{I_S}{I_D}\frac{1}{I_S} = \frac{V_T}{I_D} = \frac{KT}{qI_D}$

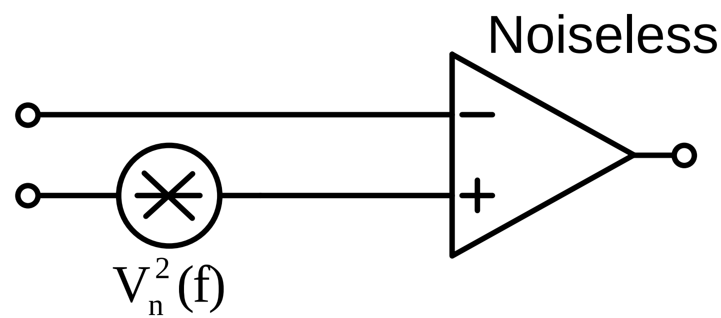
$$\Rightarrow I_d^2(f) = \frac{V_d^2(f)}{r_d^2} = 2qI_D$$

- Capacitors and inductors do not generate noise

Noise Models for Circuit Elements (Cont.)

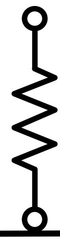
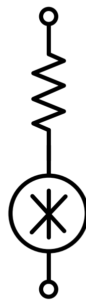
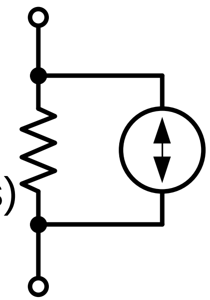
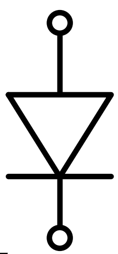
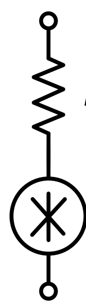
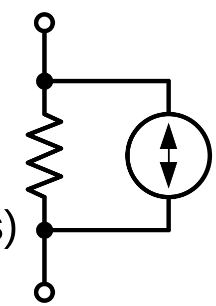
- OPAMPs
  - Bipolar OPAMP
  - CMOS OPAMP





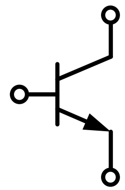
(All noise sources are uncorrelated)

Noise Models for Circuit Elements (Cont.)

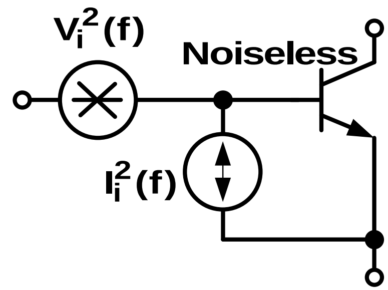
Element	Noise Models	
Resistor 	 R (Noiseless) $V_R^2(f) = 4KTR$	 R (Noiseless) $I_R^2(f) = \frac{4KT}{R}$
Diode  *(Forward Biased)*	 $r_d = \frac{KT}{qI_D}$ (Noiseless) $V_d^2(f) = 2KTr_D$	 $r_d = \frac{KT}{qI_D}$ (Noiseless) $I_d^2(f) = 2qI_D$

Noise Models for Circuit Elements (Cont.)

BJT



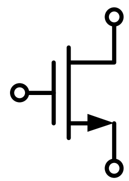
(Active region)



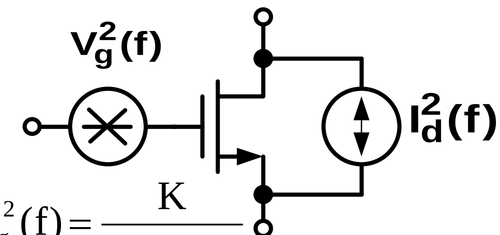
$$V_i^2(f) = 4KT\left(r_b + \frac{1}{2g_m}\right)$$

$$I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{|\beta(f)|^2}\right)$$

MOSFET

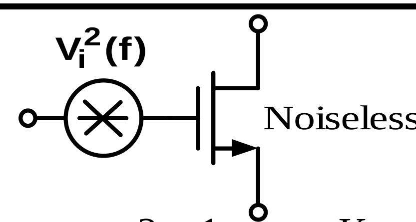


(Active region)



$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

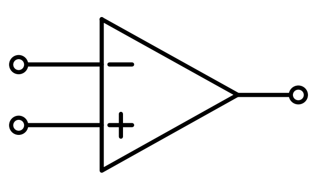
$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$$

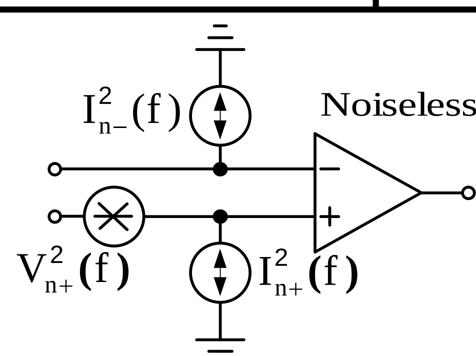


$$V_i^2(f) = 4KT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$$

simplified model for low and moderate frequencies

OPAMP



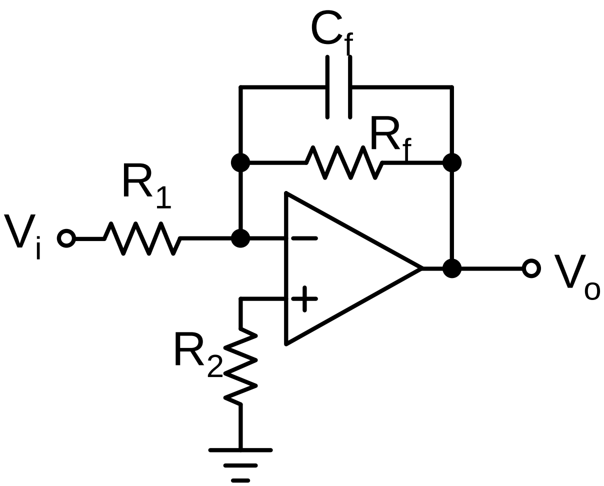


$V_n(f), I_{n-}(f), I_{n+}(f)$

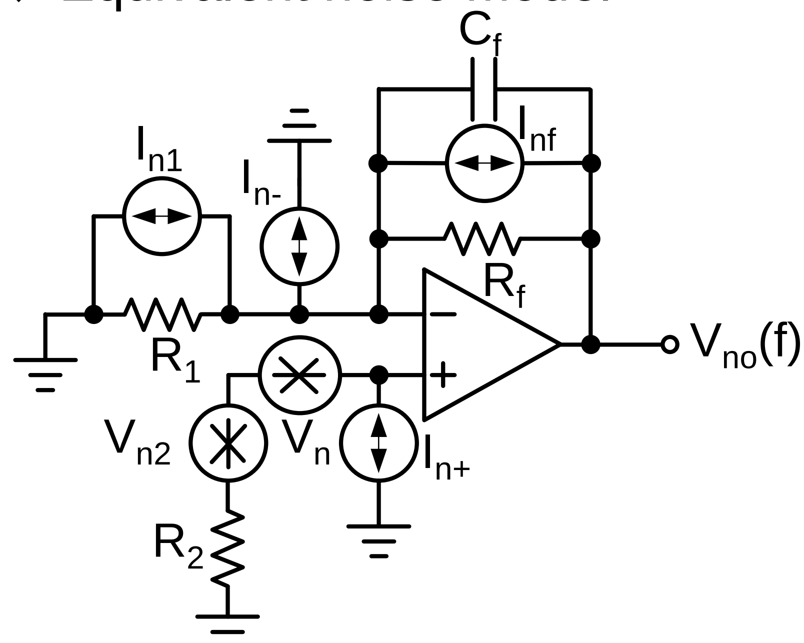
Value depends on opamp typically, all uncorrelated

Noise Analysis Examples

- OPAMP example
  - A lowpass filter



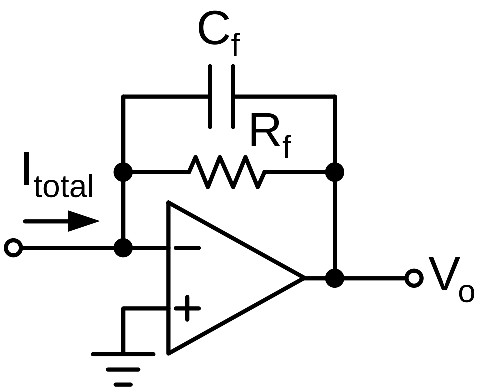
  - Equivalent noise model



  - Assuming all noise sources are uncorrelated
  - Using superposition
    - $V_{no1}^2(f)$ due to $I_{n1}(f)$, $I_{nf}(f)$ and $I_{n-}(f)$
    - $V_{no2}^2(f)$ due to $I_{n+}(f)$, $V_{n2}(f)$ and $V_n(f)$
    - $V_{no}^2(f) = V_{no1}^2(f) + V_{no2}^2(f)$

Noise Analysis Examples (Cont.)

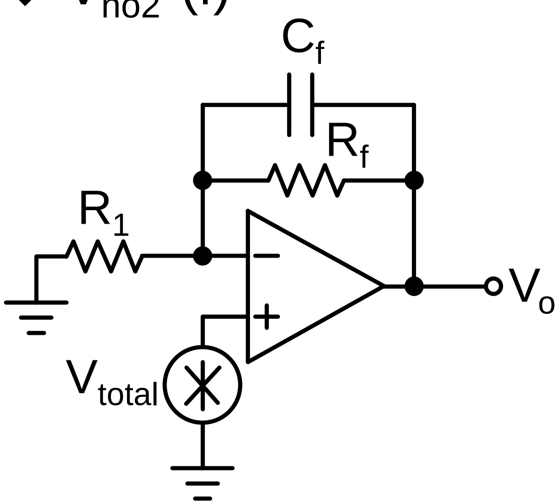
- $V_{no1}^2(f)$



$$V_o = -I_{total}\frac{1}{\frac{1}{R_f}+SC_f} = I_{total}\times\frac{R_f}{1+SR_fC_f}$$

$$V_{no1}^2(f) = \left[I_{n1}^2(f)+I_{nf}^2(f)+I_{n-}^2(f)\right]\left|\frac{R_f}{1+j2\pi fR_fC_f}\right|^2$$

- $V_{no2}^2(f)$



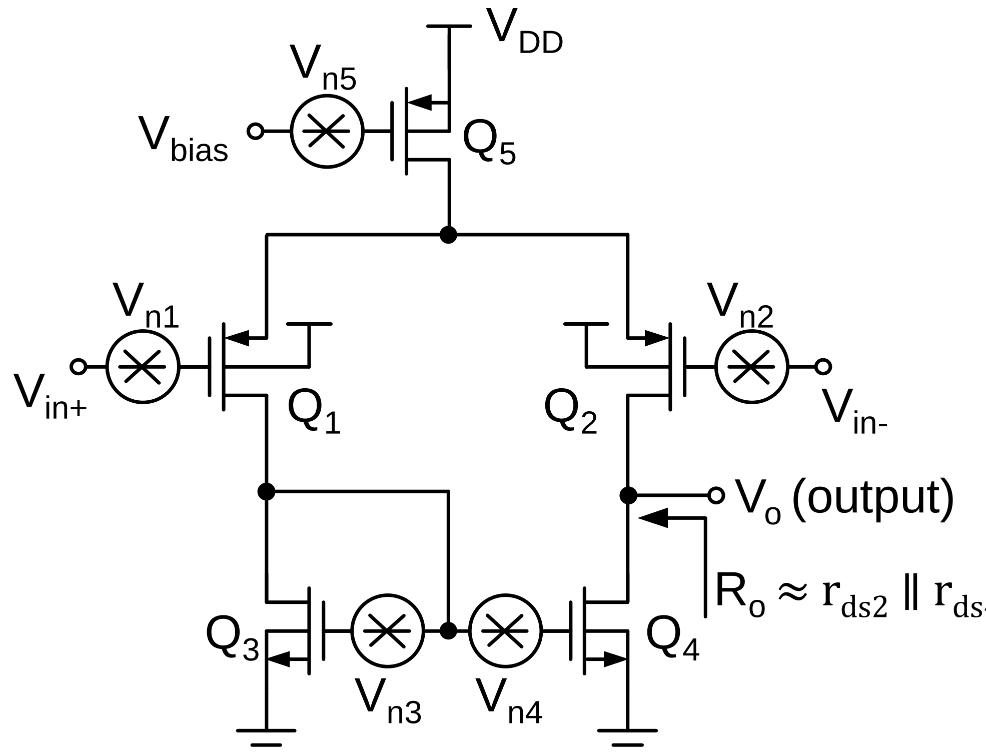
$$V_o = V_{total}\times\left(1+\frac{R_f/R_1}{1+SR_fC_f}\right)$$

$$V_{no2}^2(f) = [I_{n+}^2(f)R_2^2+V_{n2}^2(f)+V_n^2(f)]\left|1+\frac{R_f/R_1}{1+j2\pi fR_fC_f}\right|^2$$

- $V_{no(rms)}^2 = \int_0^\infty V_{no}^2(f)df = V_{no1(rms)}^2+V_{no2(rms)}^2$

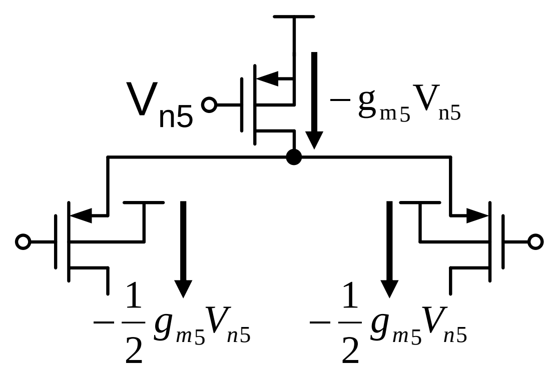
Noise Analysis Examples (Cont.)

- CMOS examples
  - A differential input stage
  - Assuming
    - Q_1 and Q_2 are identical
    - Q_3 and Q_4 are identical



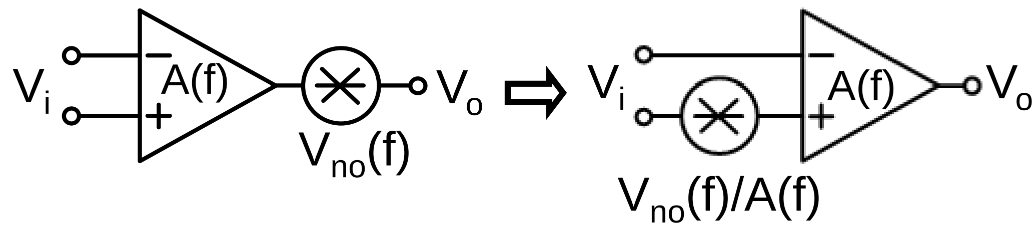
- $$\left|\frac{V_{no}}{V_{n1}}\right| = \left|\frac{V_{no}}{V_{n2}}\right| = g_{m1}R_o \qquad (1)$$
- $$\left|\frac{V_{no}}{V_{n3}}\right| = \left|\frac{V_{no}}{V_{n4}}\right| = g_{m3}R_o \qquad (2)$$
- $$\left|\frac{V_{no}}{V_{n5}}\right| = \frac{g_{m5}}{2g_{m3}} \qquad (3)$$

(The drain of Q_2 will track that of Q_1)



Noise Analysis Examples (Cont.)

- Since (3) is relative small compared to the others, it can be ignored.
  - $$V_{no}^2(f) = 2(g_{m1}R_o)^2 V_{n1}^2(f) + 2(g_{m3}R_o)^2 V_{n3}^2(f)$$
  - Equivalent input noise



$$V_{neq}^2(f) = \frac{V_{no}^2(f)}{(g_{m1}R_o)^2} = 2V_{n1}^2(f) + 2V_{n3}^2(f)\left(\frac{g_{m3}}{g_{m1}}\right)^2$$

(equivalent Input noise)

  - For the white noise portion, i.e. thermal noise

Assuming $$V_{ni(thermal)}^2(f) = 4KT\left(\frac{2}{3}\right)\left(\frac{1}{g_{mi}}\right)$$

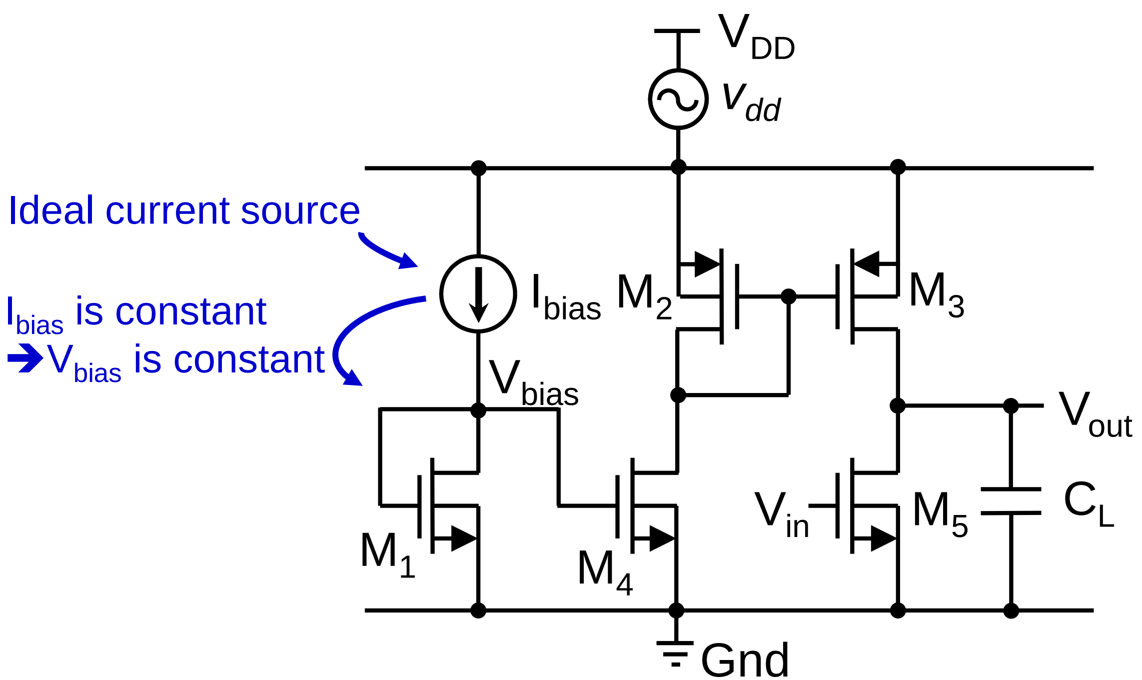
$$V_{neq(thermal)}^2(f) = \frac{16}{3}KT\left(\frac{1}{g_{m1}}\right) + \frac{16}{3}KT\left(\frac{g_{m3}}{g_{m1}}\right)^2\left(\frac{1}{g_{m3}}\right)$$

Power Supply Rejection Ratio (PSRR)

- The ratio of the differential gain Av to the gain from the power-supply ripple to the output with the differential input set to zero

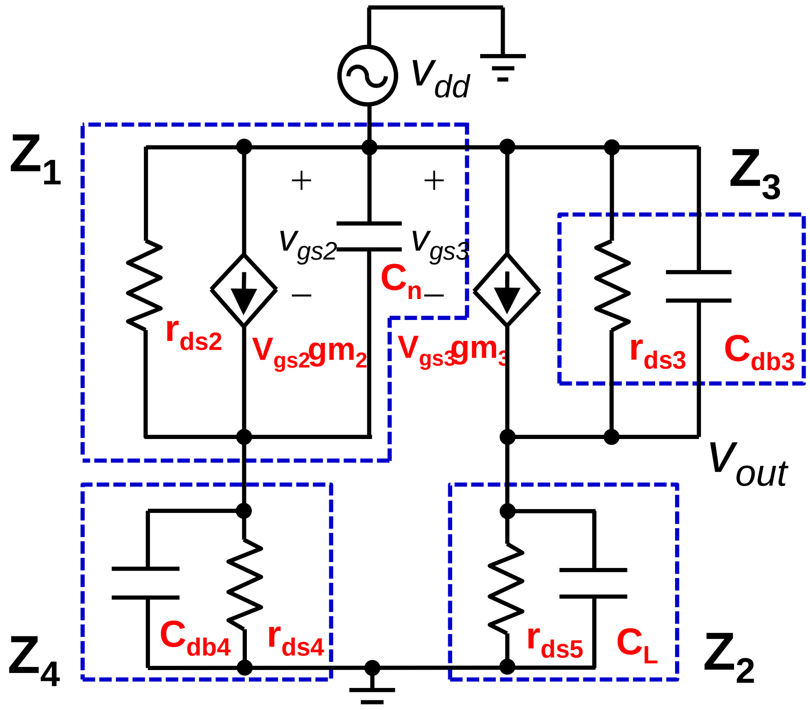
$$\mathrm{PSRR} = \frac{v_{out}/v_{in}}{v_{out}/v_{dd}}$$

- Method for calculating the PSRR of common source amplifier



Power Supply Rejection Ratio (Cont.)

- Small signal model of common source amplifier



$$C_n = C_{gs2} + C_{gs3} + C_{db2}$$

$$Z_1 = (r_{ds2} // \frac{1}{gm_2} // \frac{1}{sC_n})$$

$$Z_2 = (r_{ds5} // \frac{1}{sC_L})$$

$$Z_3 = (r_{ds3} // \frac{1}{sC_{db3}})$$

$$Z_4 = (r_{ds4} // \frac{1}{sC_{db4}})$$

- v_{out} / v_{dd} transfer function derivation

$$\begin{cases} v_{gs3} = v_{gs2} = v_{dd} \dfrac{Z_1}{Z_1 + Z_4} \\ v_{out} = v_{gs3} gm_3 (Z_3 // Z_2) + v_{dd} \dfrac{Z_2}{Z_2 + Z_3} \end{cases} \Rightarrow \begin{cases} \dfrac{v_{out}}{v_{dd}} = \dfrac{Z_1}{Z_1 + Z_4} gm_3 (Z_3 // Z_2) + \dfrac{Z_2}{Z_2 + Z_3} \\ PSRR = \dfrac{gm_5 (Z_3 // Z_2)}{\dfrac{Z_1}{Z_1 + Z_4} gm_3 (Z_3 // Z_2) + \dfrac{Z_2}{Z_2 + Z_3}} \end{cases}$$

Appendix - Noise Analysis Examples

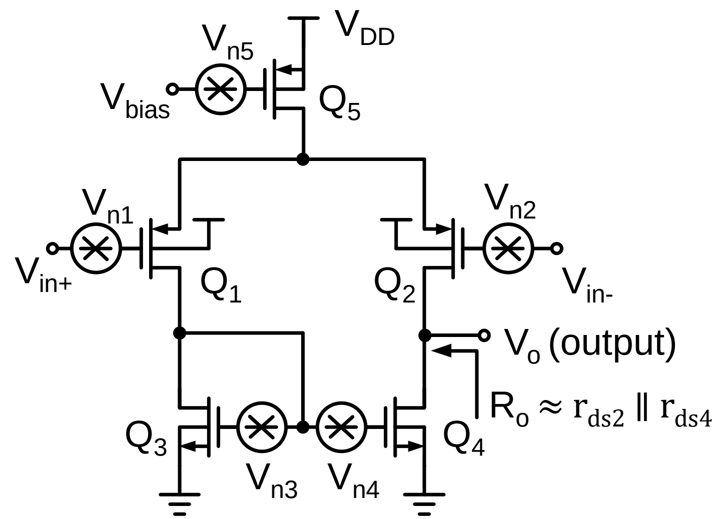
- g_{m1} should be made as large as possible to minimize
  - For flicker (1/f) noise portion

$$g_{mi} = \sqrt{2\mu_i Cox(\frac{W}{L})_i I_{Di}}$$

$$V^2_{neq(flicker)}(f) = 2V^2_{n1}(f) + 2V^2_{n3}(f)\left[\frac{(W/L)_3 \mu_n}{(W/L)_1 \mu_p}\right]$$

$$V^2_{ni(flicker)}(f) = \frac{K_i}{W_i L_i C_{ox} f}$$

$$V^2_{neq(flicker)}(f) = \frac{2}{C_{ox} f}\left[\frac{K_1}{W_1 L_1} + \left(\frac{\mu_n}{\mu_p}\right)\frac{K_3 L_1}{W_1 L_3^2}\right] \quad \ldots\ldots (9.109)\ ^1$$



[1]P.394 in the textbook

Appendix - Noise Analysis Examples (Cont.)

- Recall that the first term in (9.109)[1] is due to the p-channel input transistors, Q_1 and Q_2, and the second term is due to the n-channel loads, Q_3 and Q_4. We note some points for 1/f noise here:
  - For $L_1=L_3$, the noise of the n-channel loads dominate since $\mu_n > \mu_p$ and typically n-channel transistors have larger 1/f noise than p-channel transistors (i.e. , $K_3 > K_1$).
  - Taking L_3 longer greatly helps due to the inverse squared relationship in the second term of (9.109)[1]. This limits the signal swings somewhat, but it may be a reasonably trade-off where low noise is important.
  - The input noise is independent of W_3, and therefore we can make it large to maximize signal swing at the output.
  - Taking W_1 wider also helps to minimize 1/f noise. (Recall that it helps white noise, as well.)

[1]P.394 in the textbook

Appendix - Noise Analysis Examples (Cont.)

- Taking L_1 longer increases the noise because the second term in (9.109)[1] is dominant. Specifically, this decreases the input-referred noise of the p-channel drive transistors , which are not the dominate noise sources, but it also increases the input-referred noise of the n-channel load transistors, which are the dominant noise sources !

Total rms input noise, $V_{neq(rms.)}^2$, integrated from f_1 to f_2.

$$V_{neq(rms)}^2 = \int_{f_1}^{f_2} \left[V_{neq(thermal)}(f) + V_{neq(flicker)}(f)\right] df$$

$$= [\frac{16}{3} kT(\frac{1}{g_{m1}}) + \frac{16}{3} kT(\frac{g_{m3}}{g_{m1}})^2 (\frac{1}{g_{m3}})](f_2 - f_1)$$

$$+ 2[\frac{a_p}{w_1 L_1} + a_n (\frac{\mu_n}{\mu_p})(\frac{L_1}{w_1 L_3^2})] \;; \quad \text{where} \quad a_i = \frac{k_i}{c_{ox}} \ln \frac{f_2}{f_1}$$

[1]P.394 in the textbook